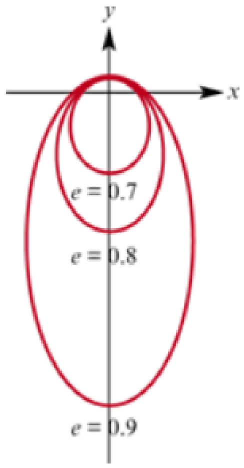
1 a

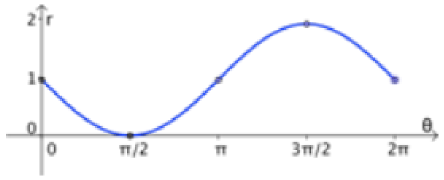


b The ellipse increases in size and becomes more narrow as e is increased.

2 a To help sketch this curve we first graph the function

$$r = 1 - \sin\theta$$

as shown below. This allows us to see how r changes as θ increases.



Note that:

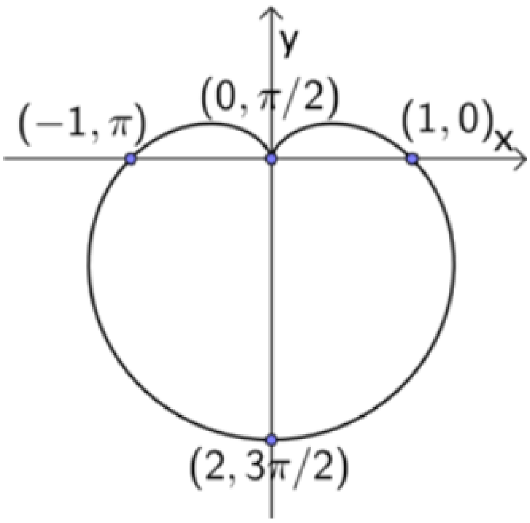
b As angle θ increases from 0 to $\pi/2$, the radius r decreases from 1 to 0.

c As angle θ increases from $\pi/2$ to π, the radius r increases from 0 to 1.

d As angle θ increases from π to $3\pi/2$, the radius r increases from 1 to 2.

e As angle θ increases from $3\pi/2$ to 2π, the radius r decreases from 2 to 1.

This gives the graph shown below. The points are labelled using polar coordinates.

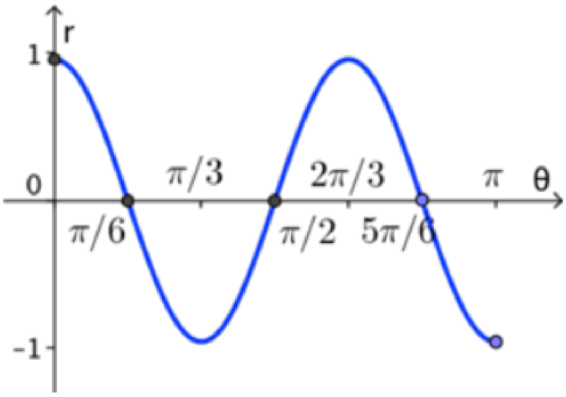


b The trick, once again, is to multiply both sides of the equation through by r. This gives,

$$\begin{aligned} r^2 &= r - r\sin\theta \\ x^2+y^2 &= r - y \\ x^2+y^2+y &= r \\ x^2+y^2+y &= \sqrt{x^2+y^2} \\ (x^2+y^2+y)^2 &= x^2+y^2, \end{aligned}$$

as required.

3 a To help sketch this curve we first graph the function $r = \cos 3\theta$ as shown below. This allows us to see how r changes as θ increases.



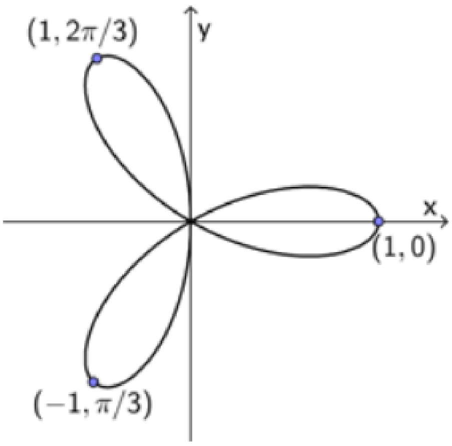
Note that:

b As angle θ increases from 0 to $\pi/6$, the radius r varies from 1 to 0.

c As angle θ increases from $\pi/6$ to $\pi/3$, the radius r varies from 0 to -1.

d As angle θ increases from $\pi/3$ to $\pi/2$, the radius r varies from -1 to 0.

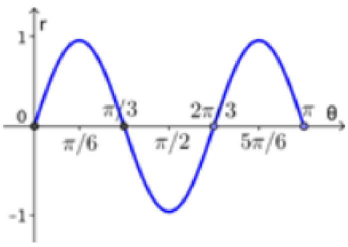
Continuing in this manner, we obtain the following graph shown below. Note that the labelled points are polar coordinates.



b To help sketch this curve we first graph the function

$$r = \cos 3\theta$$

as shown below. This allows us to see how r changes as θ increases.

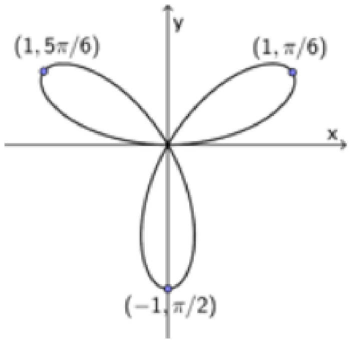


Note that:

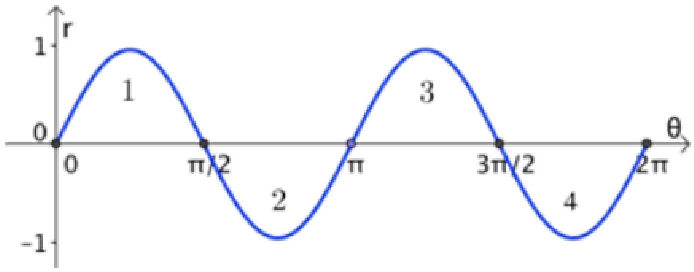
c As angle θ increases from 0 to $\pi/6$, the radius r varies from 0 to 1.

d As angle θ increases from $\pi/6$ to $\pi/3$, the radius r varies from 1 to 0.

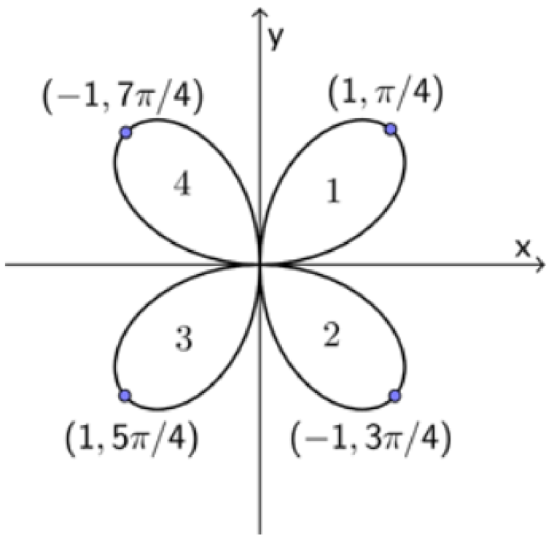
Continuing in this manner, we obtain the following graph shown below. Note that the labelled points are polar coordinates.



4 a To help sketch this curve we first graph the function $r = \sin 2\theta$ as shown below. This allows us to see how r changes as θ increases.



Using numbers, we have labelled how each section of this graph corresponds to a each section in the rose below. Note that the labelled points are polar coordinates.

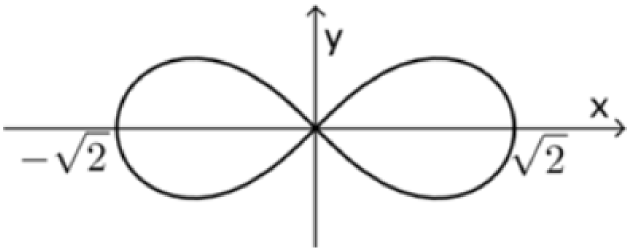


b Since $\sin 2\theta = 2\sin\theta\cos\theta$, we have

$$\begin{aligned} r &= \sin 2\theta \\ r &= 2\sin\theta\cos\theta \\ r^3 &= 2\cdot r\sin\theta\cdot r\cos\theta \\ r^3 &= 2xy \\ (x^2+y^2)^{\frac{3}{2}} &= 2xy \\ (x^2+y^2)^3 &= 4x^2y^2, \end{aligned}$$

as required.

5 a



Note that we can find the x-intercepts by letting $\theta = 0$ and $\theta = \pi$. This gives $x = \pm\sqrt{2}$.

b Since $\cos 2\theta = \cos^2\theta + \sin^2\theta$, we have

$$\begin{aligned}
r^2 &= 2\cos 2\theta \\
r^2 &= 2(\cos^2\theta - \sin^2\theta) \\
r^2 &= 2\cos^2\theta - 2\sin^2\theta \\
r^2 &= 2\left(\frac{x}{r}\right)^2 - 2\left(\frac{y}{r}\right)^2 \\
r^2 &= 2\frac{x^2}{r^2} - 2\frac{y^2}{r^2} \\
r^4 &= 2x^2 - 2y^2 \\
\left(\sqrt{x^2+y^2}\right)^4 &= 2x^2 - 2y^2 \\
(x^2+y^2)^2 &= 2x^2 - 2y^2
\end{aligned}$$

c If $d_1 d_2 = 1$ then using the distance formula, we find that

$$\begin{aligned}
\sqrt{(x-1)^2+y^2}\sqrt{(x+1)^2+y^2} &= 1 \\
[(x-1)^2+y^2][(x+1)^2+y^2] &= 1 \\
(x-1)^2(x+1)^2 + (x-1)^2y^2 + (x+1)^2y^2 + y^4 &= 1 \\
(x^2-1)^2 + [(x-1)^2+(x+1)^2]y^2 + y^4 &= 1 \\
(x^2-1)^2 + (2x^2+2)y^2 + y^4 &= 1 \\
x^4 - 2x^2 + 1 + 2x^2y^2 + 2y^2 + y^4 &= 1 \\
x^4 + 2x^2y^2 + y^4 - 2x^2 + 2y^2 &= 0 \\
(x^2+y^2)^2 - 2x^2 + 2y^2 &= 0.
\end{aligned}$$

Therefore,

$$(x^2+y^2)^2 = 2x^2 - 2y^2,$$

which is the same equation as that found previously.